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75. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy in Ohio University, Athens, Ohio.

A plane passes through $(0, 0, c)$ and touches the circle $x^2 + y^2 = a^2$, $z=0$; determine the locus of the ultimate intersections of the plane.

I. Solution by the PROPOSER.

Let the plane be $Ax + By + Cz + p = 0$ (1).

Passing through $(0, 0, c)$, (1) gives $p = -cC$ (2),

and (1) becomes $Ax + By + Cz - cC = 0$ (3).

The x, y, z of (3) are those of $x^2 + y^2 = a^2$ (4), $z=0$ (5),

and also of $Ax + By - cC = 0$ (6).

Making (4) homogeneous by aid of (6),

$$\left[\frac{1}{a^2} - \frac{A^2}{c^2 C^2} \right] \frac{x^2}{y^2} - \frac{2AB}{c^2 C^2} \frac{x}{y} + \left[\frac{1}{a^2} - \frac{B^2}{c^2 C^2} \right] = 0 \text{ (7).}$$

For (3) to touch (7), the values of x/y from (7) must be equal, or

$$\left[\frac{1}{a^2} - \frac{A^2}{c^2 C^2} \right] \left[\frac{1}{a^2} - \frac{B^2}{c^2 C^2} \right] = \frac{A^2 B^2}{c^4 C^4} \text{ (8),}$$

$$\text{or, } A^2/C^2 + B^2/C^2 - c^2/a^2 = 0 \text{ (9).}$$

From (3), $A/C = (z-c)/x - (y/x)(B/C)$ (10).

Substituting (10) in (9), etc.,

$$\frac{x^2 + y^2}{x^2} B^2/C^2 - \frac{2y(z-c)}{x^2} B/C + \frac{(z-c)^2}{x^2} - c^2/a^2 = 0 \text{ (11),}$$

a quadratic in the undetermined constant B/C , giving the envelope

$$\frac{x^2 + y^2}{a^2} = \frac{(z-c)^2}{c^2} \text{ (12).}$$

II. Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas.

Let the plane touch the circle at the point (x', y') .

$\therefore \frac{xx'}{a^2} + \frac{yy'}{b^2} + z/c = 1$, is the equation to the plane, but

$$x'^2 + y'^2 = a^2 \text{ (1). } \therefore dy'/dx' = -x'/y' = -x'/y' \text{ (2).}$$

(2) in the equation to the plane gives,

$$x' = \frac{a^2 x(c-z)}{c(x^2+y^2)}, \quad y' = \frac{a^2 y(c-z)}{c(x^2+y^2)}.$$

These values of x' , y' in (1), $x^2 + y^2 - (a^2/c^2)(c-z)^2 = 0$, a cone of revolution as the locus.

MECHANICS.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

49. Proposed by O. W. ANTHONY, M. Sc., Professor of Mathematics, Columbian University, Washington, D. C.

A rectangular stick of timber of known dimensions is placed upon a platform of given height in a vertical position with the center above the edge of platform, and slightly displaced from the vertical. Where and in what manner will it strike the ground?

I. Solution by the PROPOSER.

Any body rotating about the center of an end has the energy,

$$\frac{1}{2}(\omega^2)m(a^2 + b^2).$$

If the body has fallen through the angle θ , the energy is

$$\frac{1}{2}b(1 - \cos\theta)m.$$

$$\therefore 2\omega^2(a^2 + b^2) = 3b(1 - \cos\theta) + \dots \dots \dots (1).$$

The body will leave the platform when the statical pressure = centrifugal force. The pressure = $m\cos\theta$. Centrifugal force = $\frac{1}{2}m\omega^2 b$.

$$\therefore \omega^2 b = 2\cos\theta \dots \dots \dots (2).$$

$$\text{From (1) and (2), } \omega = \sqrt{\frac{6b}{4a^2 + 7b^2}} \dots \dots (3), \text{ and } \cos\theta = \frac{3b^2}{4a^2 + 7b^2} \dots \dots (4).$$

Take the edge of platform as origin. Let the axis of x be horizontal and the axis of y vertical. Resolve the angular velocity of the center of gravity into its vertical and horizontal components at the instant of the stick leaving the platform.

$$\left. \begin{aligned} V_x &= \frac{1}{2}b\omega\cos\theta \\ V_y &= \frac{1}{2}b\omega\sin\theta \end{aligned} \right\} \dots \dots \dots (5).$$